A Subtlety of Earth's Gravity and its Rotation

David Alexander Lillis, 9 March 2019

This article provides an elementary introduction to the effect of Earth's non-spherical shape (oblateness) and the centrifugal force on Earth's gravity for non-Earth scientists, students and other interested readers. In this article I describe a simple model of the gravitational acceleration on Earth's surface, taking into account both the gravitation of the Earth itself and the effect of centrifugal forces acting on bodies located on Earth's surface. This model was developed purely for instructional purposes and is not intended to supplant more accurate and much more complex models already available in the relevant literature. Software written in the R language can be provided for those who wish to reproduce the calculations and graphs presented here.

Part 1: Weight involves a combination of Gravity and Centrifugal Effect

As is well known, Earth is not a perfect sphere. Because of its rotation the centrifugal effect makes it bulge slightly at the equator and slightly squashed at the poles, to produce what we refer to as an 'oblate spheroid'. In fact, its observed diameter is approximately 43 km greater at the equator than at the poles, leading to slightly stronger gravitation at the poles than at the equator. However, the centrifugal effect also applies to a body on the Earth's surface as it rotates about Earth's rotation axis. This force appears to modify the effect of pure gravity and tends to reduce the quantity we call 'weight'.

Figure 1 shows both the gravitational force and the centrifugal force acting on a body at Earth's surface at different latitudes, and their resultants (the vector sums of the gravitational forces and the centrifugal forces).

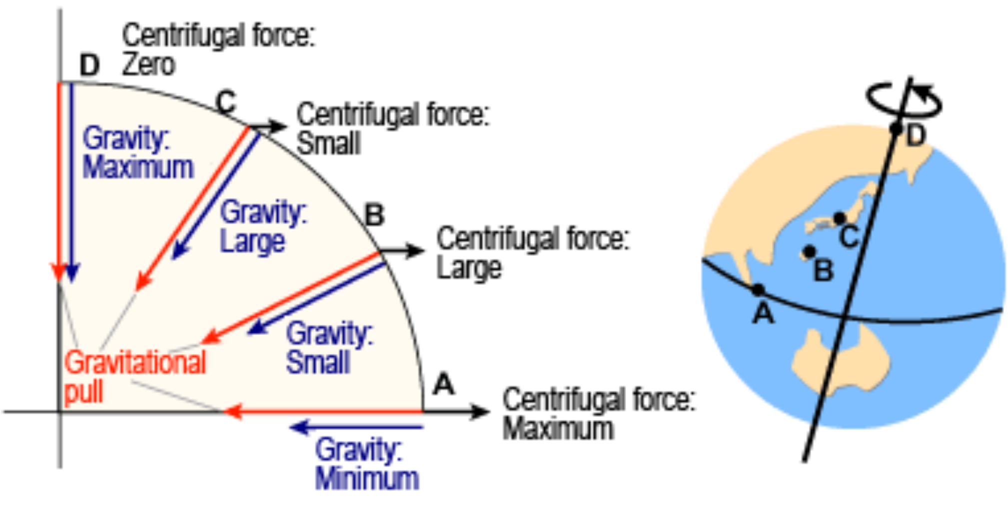

Figure 1: Gravitational force, centrifugal force and their resultants at different latitudes

In Figure 1 (Shimadazu, 2019) the Earth's gravitational force is represented by red arrows that point from Earth's surface directly toward Earth's centre. The red arrow lengths represent the strength of the gravitational force at various latitudes. In fact, pure gravity (not considering the centrifugal force) is slightly stronger at the poles than elsewhere (especially at the equator), mainly because the surface there is slightly closer to the Earth's centre of mass than other regions of the Earth's surface. In addition, the centrifugal force is weakest at the poles and strongest at the equator, so that a body appears to weigh even more at the poles than at the equator. Thus, the centrifugal effect appears twice in our present context, both giving rise to Earth's oblateness and acting on bodies on the rotating Earth's surface.

In the context of a body on the Earth's surface, the centrifugal effect is identical to the effect we get when we rotate a small mass attached to one end of a piece of string. In Figure 1 the black arrows indicate schematically the direction and magnitude of the centrifugal effect at various latitudes. The effect is vertical and greatest at the equator because any point on the equator moves around the Earth's axis of rotation at greater speed than at other latitudes. Thus, gravity appears to be slightly reduced near the equator. In the regions of the poles the centrifugal effect is very small and is nearly horizontal. Here, gravity dominates the centrifugal force.

The dark blue arrows indicate the resultants of vector addition of the gravitational forces and centrifugal forces (i.e. the resultant of local gravity and the centrifugal forces). The combination of slightly reduced gravitation at the equator and the significant vertical centrifugal force means that the apparent force of gravity at the equator and the apparent weight of a body are approximately 0.5% less than at the poles.

Part 2: Variation of Speed on the Rotating Earth

The angular velocity of the Earth as it spins is 360° or 2π radians per day which gives 7.292×10^{-5} rad/s or approximately 0.00417° per second. The Earth's angular velocity is fixed for the entire Earth. However, the linear velocity of the Earth's surface is much greater at the equator than at the poles. At the equator the surface moves eastward at almost 1,674 km/hr (465 m/s), but at approximately 0.00008 km/hr (2.22×10^{-5} m/s) at the poles. In general, the linear speed V(φ) at Earth's surface at a given latitude φ is approximately:

$$V(\varphi) = 1670 \cos\varphi \text{ km/hr} \quad = \quad 463.89 \cos\varphi \text{ m/s}$$

. . . where the latitude φ is the angle subtended from the plane of the equator to the chosen point on Earth's surface.

Part 3: A Schematic Diagram of the Earth and its Radius at a given Latitude

Figure 2 shows the rotating Earth, latitude (φ) - the angle subtended from Earth's centre to a point on Earth's surface at point P, Earth's radius r for latitude φ (varying because of its centrifugally-induced oblateness) and the equatorial and polar radii. Here, Earth's radius r is the radial distance from Earth's centre to the chosen point on the Earth's surface at the given latitude.

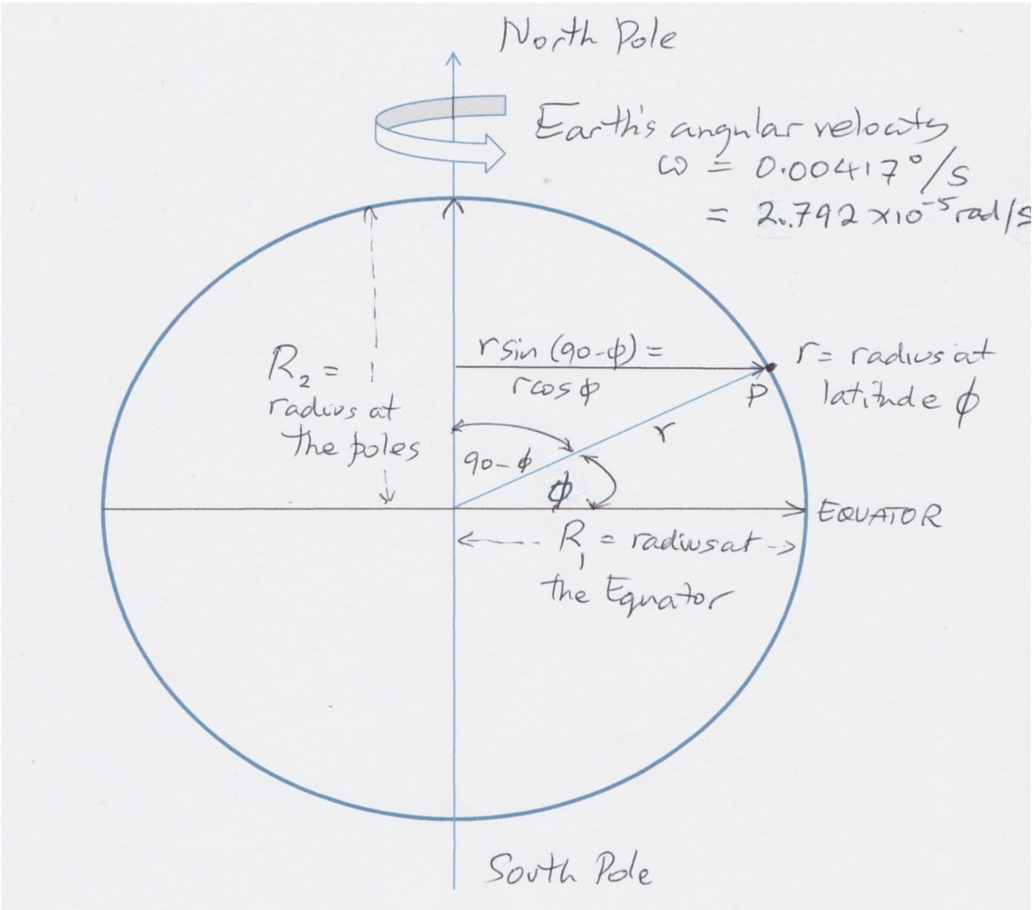

Figure 2: The Earth, a point P on Earth's surface at a given latitude φ, Earth's radius r for latitude φ and the equatorial and polar radii.

In Figure 2 the Earth rotates at angular velocity ω (for the purposes of the calculations described in this article we assume the value for ω in radians given above - approximately 7.292×10^{-5} rad/s). For a point P on the Earth's surface, situated at latitude φ and Earth's radius r from Earth's centre, the radial distance from the axis of rotation to the chosen point P is:

$$r\sin(90^{o} - \varphi) = r\cos\varphi.$$

This quantity gives the local circumference of the Earth at the given latitude. Of course, the apparent gravitational acceleration varies slightly by latitude, both because of Earth's oblateness and the centrifugal force. The R code of Appendix 1 enables calculation of Earth's radius due to oblateness at any latitude, the local radial distance and ground speed doe to rotation.

Part 4: Earth's Gravitational and Centrifugal Accelerations at a given Latitude

Figure 3 shows the Earth's gravitational acceleration g(φ), pointing towards the Earth's centre, the centrifugal acceleration resolved into radial and tangential components and the vector sum (resultant) of the two (g_{eff}).

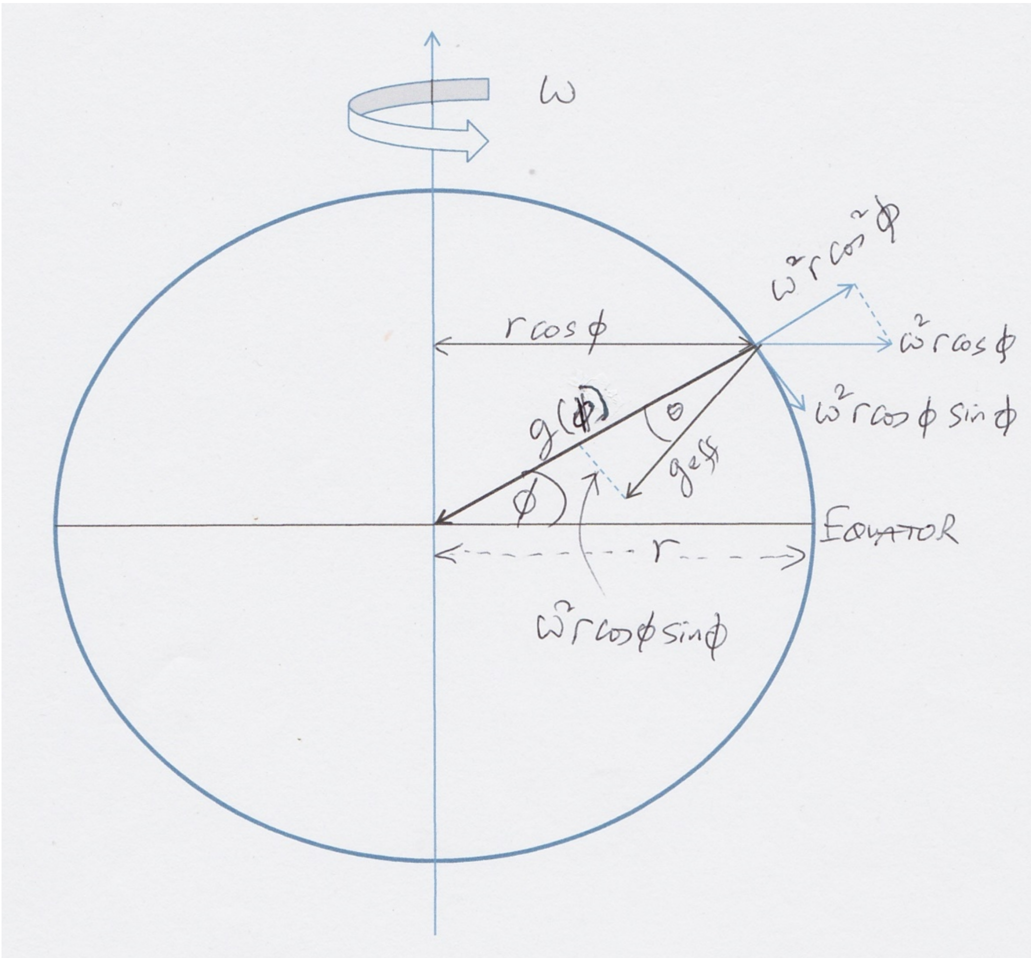

Figure 3: Gravitational force acting towards the Earth's centre, the centrifugal force at a given latitude and the vector sum (resultant) of the two forces.

We refer to the resultant as effective gravity (g_{eff}). The resultant involves a slight reduction in the apparent weight of a body as we move from the poles to the equator and the resultant no longer points towards the Earth's centre. As before, r is Earth's radius at the given latitude – the distance from the Earth's centre to the point on the Earth's surface at the given latitude φ. The angle Θ is the angle between g(φ) and the resultant g_{eff}.

Part 5: Setting up our Model

In the calculations of this article (and associated R software) we calculate g(φ) for the given latitude and its corresponding Earth radius on the basis of Newton's Law of Gravitation:

$$g(\varphi) = G\ M\ /\ R(\varphi)^2 \qquad \text{. . . equation 1}$$

. . . where G is the Gravitational Constant (6.674×10^{-11} m^3 $kg^{-1}s^{-2}$) and M is the Mass of the Earth (5.972×10^{24} kg).

Though it is not exact, we use this Newton-based approach for g(φ) rather than the conventional standard value of 9.80665 m/s^2 (CPGM, 2019). This single value is approximately true for the entire Earth, ignoring the small effect of oblateness, but Newton's approach reflects an actual variation in gravitational acceleration across latitudes.

In fact, the increase in g(φ) due to oblateness is approximately 0.1 as large as expected from an inverse square law, and is approximately 0.33 of the observed value (Iona, 1978). The observed value is also larger because the Earth's density increases toward the center. Thus, our calculated model values for local gravitational acceleration slightly overestimate the true values. For example, our calculated value of g(φ) at the poles is 9.86 m/s^2, whereas the measured value is 9.83 m/s^2. In our simple Newton-based model, g(φ) is overestimated by approximately 0.2% at the poles and by approximately 0.3% at the equator.

We see the resultant of g(φ) and the centrifugal force in reducing apparent gravity and offsetting the apparent gravitational force away from Earth's centre (shown here by the angle ϴ). In fact, ϴ is very small and the schematic diagram of Figure 3 magnifies the angle considerably, simply for clarity of the diagram. On Earth, the tangential component is small and ϴ is largely cancelled out by friction, but can be useful to consider the tangential component in applications such as analysis of the dynamics of oceans and the atmosphere.

Part 6: Calculating the Magnitude of Effective g

We now attempt to quantify the apparent reduction in g(φ) and determine by how much g_{eff} deviates from pointing towards Earth's centre. Now, the centrifugal acceleration A at the chosen point on Earth's surface is:

$$A = V^2 / \text{local radial distance at latitude } \varphi = V^2 / r \sin(90 - \varphi) = V^2 / r \cos \varphi$$

. . . where V is the speed of rotation of the chosen point on Earth's surface and r is Earth's radius at latitude φ. The local radius is measured directly from the Earth's rotation axis, rather than from Earth's center, and is not to be confused with the actual Earth's radius at that latitude. Since V is the product of radial distance from Earth's rotation axis and the angular velocity ω, the centrifugal acceleration can be written:

$$(r \cos \varphi)^2 \omega / r \cos \varphi = \omega^2 r \cos \varphi \quad \ldots \text{ equation 2}$$

Of course, the centrifugal force on a body of mass M is simply M $\omega^2 r \cos \varphi$.
Note that these formulae apply to both the northern hemisphere and southern hemisphere because the cosine function is symmetric about the angle zero (in our present context zero represents the equator). Even if we assume latitudes in the southern hemisphere to have negative values of φ, these expressions will give the same result as for the equivalent latitudes in the northern hemisphere.

In Figure 3 we see that the purely gravitational acceleration $g(\varphi)$ is directed towards the Earth's centre. So, to combine the gravitational acceleration with the centrifugal acceleration in order to calculate the resultant, we resolve the centrifugal acceleration into two components: one tangential to the Earth's surface and another component normal to the Earth's surface (and therefore anti-parallel with the local purely gravitational acceleration). These components are as follows:

1. The normal component of centrifugal acceleration = $\omega^2 r \cos^2 \varphi$ (i.e. an extra factor of $\cos \varphi$ appears when we resolve the centrifugal acceleration normal to Earth's surface).
2. The tangential component of centrifugal acceleration = $\omega^2 r \cos \varphi \sin \varphi$

Thus, the vector sum of the local gravitational acceleration and the centrifugal acceleration appears to reduce $g(\varphi)$ and make it point slightly away from the Earth's centre. The normal component of centrifugal acceleration effectively reduces $g(\varphi)$ by the quantity $\omega^2 r \cos^2 \varphi$, while the tangential component also modifies $g(\varphi)$, tending to increase its magnitude by a very small amount (except at the poles), and offsets g_{eff} from Earth's centre by the very small amount $\omega^2 r \cos \varphi \sin \varphi$.

Thus, to find the magnitude of the resultant g_{eff} we use the Pythagorean expression:

$$g_{eff} = \{ (g(\varphi) - \omega^2 r \cos^2 \varphi)^2 + (\omega^2 r \cos \varphi \sin \varphi)^2 \}^{\mathbf{0.5}} \quad \ldots \text{ equation 3}$$

How significant is the effect of centrifugal force in apparently reducing $g(\varphi)$ and offsetting it from Earth's centre? Let's perform some calculations and, as an example, take our latitude to be 40° north, so that $\varphi = 40°$. In radians: $\varphi = 40 * \pi / 180 = 0.698$ radians. Of course, for the equator $\varphi = 0°$, while for the poles $\varphi = 90°$.

Because of oblateness, Earth's radius at a latitude of 40° is slightly different from the equatorial and polar radii. At latitude 40° Earth's radius is approximately: 6369.345 km. We can calculate this value using the following formula (equation 4):

$$R(\varphi) = \{ [(R_1^2 \cos(\varphi))^2 + (R_2^2 \sin(\varphi))^2] \ / [(R_1 \cos(\varphi))^2 + (R_2 \sin(\varphi))^2] \}^{0.5}$$

. . .where R_1 is the accepted radius at the equator (6378.137 km) and R_2 is the accepted radius at the poles (6356.752 km). For a proof of equation 4 see Planetcalc (2019).

Also, at a latitude of 40° the local circumference of the Earth (measured from the rotation axis, rather than from Earth's centre) is:

$$2\pi \text{ (Earth's local radius in the plane of the given latitude)}$$
$$= 2\pi \text{ (Earth's actual radius at the given latitude)}.\cos\varphi$$
$$= 2\pi \text{ (6369.345 km)}.\cos\varphi = 30{,}656.93 \text{ km}$$

The speed of rotation of the surface about the axis at latitude 40° is therefore:

$$30{,}656.93 \text{ km} / 24 \text{ hours} = 1{,}277.372 \text{ km/hr or } 354.826 \text{ m/s.}$$

To calculate the Earth's radius at a given latitude using the above formula for $R(\varphi)$ I wrote a short piece of code in the R language for statistics and graphics (please see Appendix 1). The code of Appendix 1 also calculates the local circumference at a given latitude (as before - measured from the rotation axis to the chosen point on Earth's surface in the plane of the given latitude) and the speed of the Earth's surface (due to Earth's rotation) at the given latitude. Of course, the R code of Appendix 1 enables these calculations to be undertaken for any latitude.

Effective g (g_{eff}) is now the resultant (Pythagorean sum) of actual $g(\varphi)$ minus the normal component of centrifugal acceleration (ω^2 r $\cos^2 \varphi$) radially and including the tangential component of the centrifugal acceleration ω^2 r cos φ sin φ. Our model predicts g_{eff} for latitude 40° at 9.805 m/s^2.

The code of Appendix 2 calculates g_{eff} at a given latitude φ using equation 3. The code predicts $g(\varphi = 0)$ = 9.80 m/s^2 for the equator (approximately 0.2 m/s^2 higher than the observed value) and g_{eff} for the equator at 9.764 m/s^2. The code also predicts the polar $g(\varphi)$ = 9.864 m/s^2 (approximately 0.3 m/s^2 higher than the observed value) and g_{eff} for the poles also at 9.864 m/s^2 because the centrifugal force does not diminish $g(\varphi)$ there.

We have seen that Newton's Law is an imperfect approximation for $g(\varphi)$. In fact, the increase in $g(\varphi)$ resulting from oblateness is less than that expected from a purely inverse square law, and predicts approximately one third of the observed value for Earth, partly because the Earth's density increases toward the center. Thus, our calculated values for local gravitational acceleration g(φ) slightly overestimate the observed values.

Part 7: Calculating the Direction of Effective g

The code of Appendix 3 calculates Θ, the angle between g(φ) which points directly to Earth's centre, and g_{eff} at a given latitude. We calculate Θ from the inverse tan of the tangential and normal components of the purely gravitational g(φ) and centrifugal accelerations, as follows:

$$\Theta = \tan^{-1}\{ \omega^2 \ r \cos \varphi \sin \varphi \ / \ (g(\varphi) - \omega^2 \ r \cos^2 \varphi) \} \quad \ldots \quad \text{equation 5}$$

The calculated angle between g(φ) and g_{eff} at latitude 40° is 0.097°. In other words, the angle of apparent deflection of the pure gravitational force g(φ) is small but measureable. As expected, the model predicts for both the equator (φ = 0°) and the poles (φ = 90°) an angle of zero between g(φ) and g_{eff} . These results agree with the details of Figure 1.

In our model, the angle Θ reaches its maximum at latitude 45°, where it takes the value 0.0975°. Thus, the angle between g(φ) and g_{eff} is noticeable in the mid-latitudes, but is close to zero in the vicinity of both the equator and the poles.

Part 8: Variation of Earth's Radius, g(φ), Effective g and Theta with Latitude

In this section we consider briefly some simple models of the variation of Earth's radius and g(φ) by latitude, radial and g_{eff} by latitude and the variation of Θ by latitude. Figure 4 gives the variation of Earth's radius by latitude φ, due simply to oblateness of the Earth. The R code to produce this graph is given in Appendix 4.

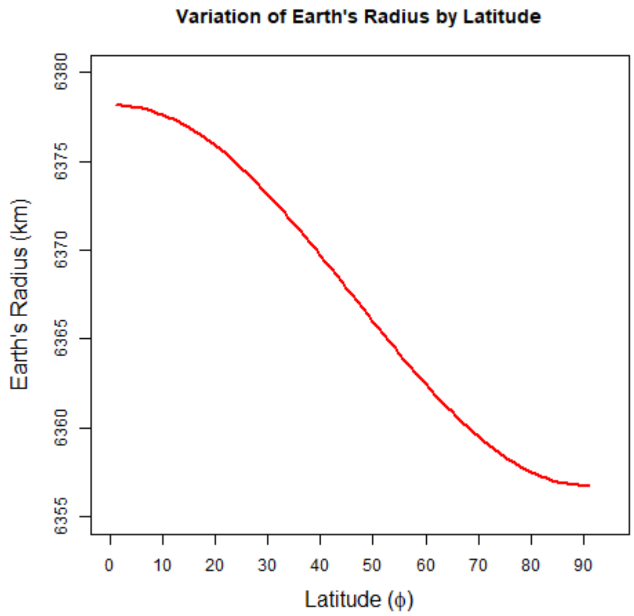

Figure 4: Variation of Earth's Radius by Latitude φ

The code used to calculate the variation of Earth's radius by latitude φ involves the Pythagorean expression of equation 3, but evaluated over the range of latitudes from the equator to the poles. The model assumes a minimum Earth's radius of 6356.752 km at the poles and a maximum radius of 6378.137 km at the equator. The difference is 21.385 km. The graph of Figure 4 models the radius for the northern hemisphere (where φ varies from 0^o to 90^o), but the results are identical for the southern hemisphere where φ varies from -90^o to 0^o.

Figure 5 gives the variation of purely gravitational acceleration g(φ) by latitude φ, resulting from our Newton-based model (equation 1) and neglecting the centrifugal force (i.e. variation in acceleration due simply to oblateness of the Earth). The R code to produce this graph is given in Appendix 4.

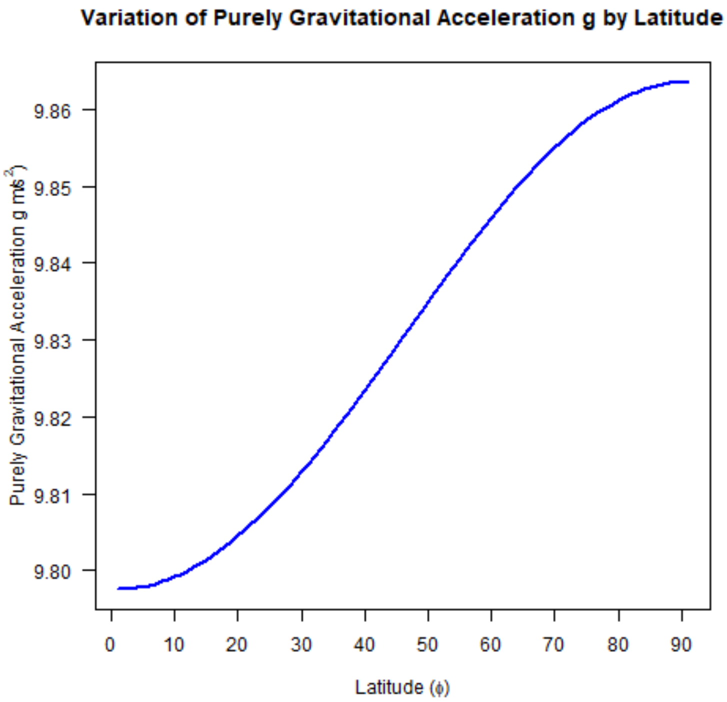

Figure 5: Variation of purely gravitational acceleration g(φ) by latitude φ.

As stated earlier, the true increase in g(φ) due to oblateness is approximately 0.1 as large as expected from an inverse square law, and is approximately 0.33 of the observed value (Iona, 1978). Thus our model slightly overestimates g(φ).

Figure 6 gives the variation of radial g and g_{eff} by latitude φ, due both to oblateness and the centrifugal force at the Earth's surface. We define radial g as the sum of g(φ) and the radial component of the centrifugal acceleration (i.e. neglecting the small tangential component), as follows:

$$\text{radial } g = g(\varphi) - \omega^2 r \cos^2 \varphi \quad \ldots \text{ equation 6}$$

In fact, both quantities (radial g and g_{eff}) are almost exactly the same because the tangential component of the centrifugal acceleration is so small. The R code producing this graph is also given in Appendix 4.

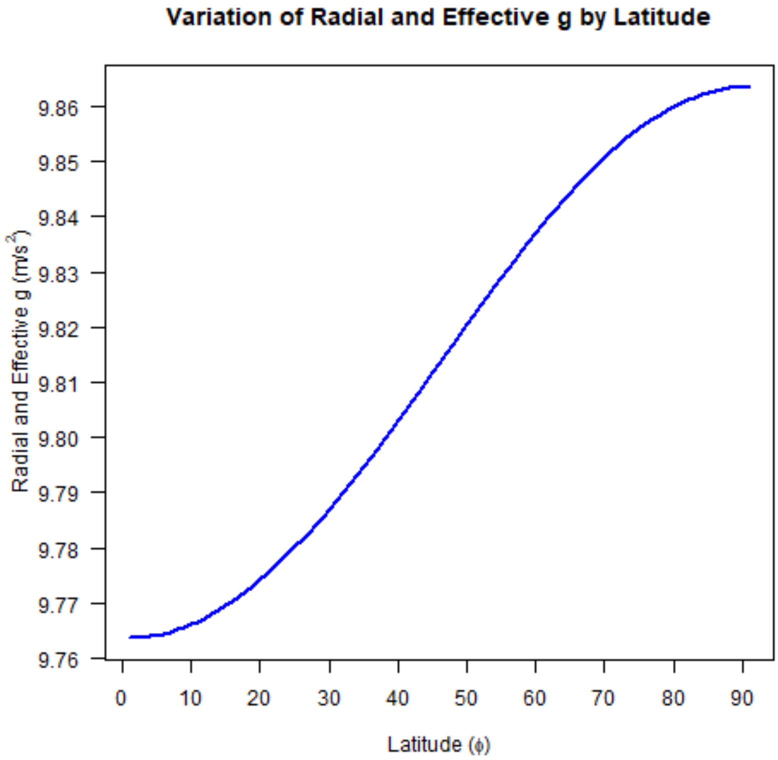

Figure 6: Variation of radial g and effective g by Latitude φ

The model of Figure 6 is nearly, but not quite, identical to that of Figure 5. Our simple model predicts a minimum g_{eff} and radial g of 9.764 m/s^2 at the equator and a maximum of 9.864 m/s^2 at the poles. Thus, a 70 kg person will appear to weigh 70 kg X 9.764 m/s^2 = 683.48 kg m/s^2 at the equator, but 70 kg X 9.864 m/s^2 = 690.48 kg m/s^2 at the poles. The difference is 7.0 kg m/s^2 (apparently a little heavier at the poles). Thus, in our model, which slightly overestimates the effect of oblateness on g(φ), the person will appear to weigh approximately 1% more at the poles. The true difference is approximately 0.5%

Figure 7 gives the variation of Θ (the angle between g and g_{eff}) by latitude φ, due to the centrifugal force at the Earth's surface. The model assumes the expression for Θ given in equation 5.

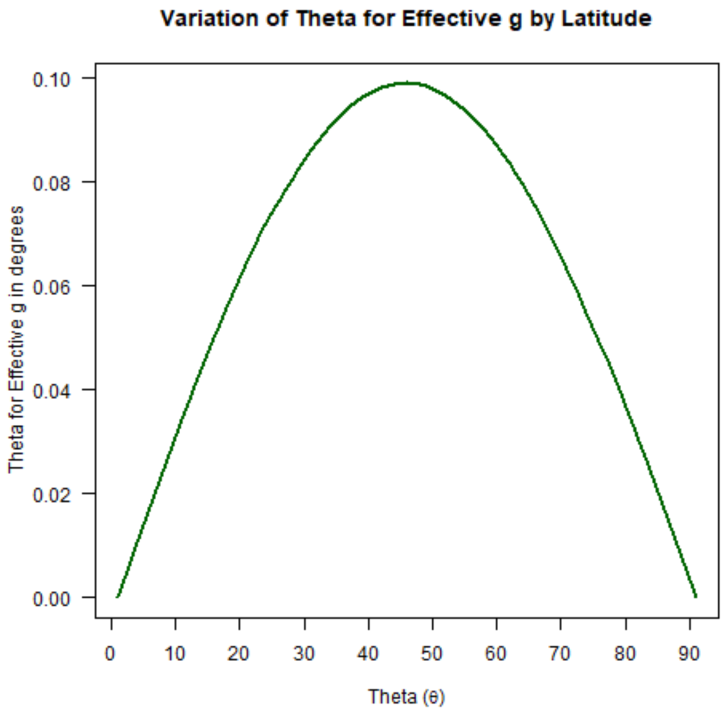

Figure 7: Variation of Θ by latitude φ

The model predicts a minimum Θ of zero at both the equator and the poles. It also predicts a maximum of 0.099° at latitudes 45° in both the northern and southern hemispheres. The R code to produce this graph is also given in Appendix 4.

Part 9: Other Complicating Factors

We observe slightly greater acceleration and therefore greater apparent weight at the poles, resulting from both centrifugal forces (which are close to zero at the poles and therefore do not diminish the effect of pure gravity there) and the small but measureable effect of oblateness (itself a result of the centrifugal effect). Both effects are at their maxima close to latitudes 45° (north and south). Finally, terrestrial mountains, sea mounts, cities and rock of varying densities all have their own small but measureable effect on g_{eff}.

Part 10: Further Reading

Gravitation is a vibrant research field at the professional level and much interesting material is available in relevant geophysical and other journals. However, a considerable volume of relevant material is available on the Internet. Simply Google 'Earth gravity and centrifugal force', or 'International Gravity Formula' or use other sensible key words to download relevant material. For example, you will find that the International Gravity Formula provides a range of formulae for the variation of gravity with latitude. For example, a commonly-used variant of the International Gravity Formula is, as follows:

$$g(\varphi) = 9.780327[\ 1 + 0.0053024 \sin^2\varphi - 0.0000058 \sin^2(2\varphi)\ ]\ m/s^2$$

. . . where the constant 9.780327 is a fitted model value of $g(\varphi = 0)$ at the equator. However, many other formulae are used to compute $g(\varphi)$. Frequently, such models involve fitting functions to the observed apparent gravitational acceleration, taking account of both oblateness and the centrifugal force at the same time. Such models would be less useful than the approach taken in this article, because the separate effects of purely gravitational acceleration and centrifugal acceleration can no longer be isolated.

References

CPGM (2019). *Standard Gravity.* Retrieved from https://planetcalc.com/7721/

Iona, M. (1978). *Why is g larger at the poles?* American Journal of Physics 46, 790 (1978).

Planetcalc (2019). *Earth Radius by Latitude.* Retrieved from https://planetcalc.com/7721/

Shimadazu (2019). *Error Caused by Difference in Gravity at Different Latitudes.* Retrieved from https://www.shimadzu.com/an/balance/support/hiroba/bean/bean06.html

Note

The following appendices present code written in the R language for calculation of Earth's radius, purely gravitational $g(\varphi)$, effective g and ϴ. I am happy to provide the code in a text file to anyone who wishes to use it. To obtain the code please e-mail me at: sigma@outlook.co.nz

APPENDIX 1

R code for calculation of Earth's radius at a given latitude, variation in g by latitude, local circumference and local speed due to Earth's rotation

Instructions: Create a folder to store the pdf graphs of Appendix 4 and navigate your R working directory to that folder. Copy and paste all of the code below into the R workspace together.

```
rm(list=ls())  # Clears the R workspace

R1 <- 6378.137  # Established radius in km at the equator

R2 <- 6356.752  # Established radius in km at the poles

phideg <- 40 # I have used 40 degrees here, but enter any value of latitude
as an angle in degrees, simply by replacing the 40 in this line of code

phirad <- phideg * pi/180 # Automatic conversion from degrees to radians,
as required by R
```

```
NUMERATOR <- (R1**2 * cos(phirad))**2 + (R2**2 * sin(phirad))**2

DENOMINATOR <- (R1 * cos(phirad))**2 + (R2 * sin(phirad))**2

RADIUS <- (NUMERATOR / DENOMINATOR)**0.5

RADIUS

LOCALCIRCUMFERENCE = 2 * pi * RADIUS * cos(phirad) # Calculates the
circumference in km at the given latitude

LOCALCIRCUMFERENCE

LOCALVELOCITY = LOCALCIRCUMFERENCE / 24  # Speed in km per hour

LOCALVELOCITY

LOCALVELOCITYMS = LOCALVELOCITY*1000/3600 # Speed in m per second

LOCALVELOCITYMS
```

APPENDIX 2

R code for calculation of the magnitude of radial g and effective g at a given latitude. Also calculates radial and effective weight for a given body mass in kg.

Instructions: Copy and paste all of the code below into the R workspace together.

```
rm(list=ls()) # Clears R workspace

# RADIUS    <-  6378.137  # Reminder of radius at equator.

# RADIUS    <-  6356.752  # Reminder of radius at poles.


W  <-  7.292e-5  # Angular velocity of Earth in radians per second

RADIUS  <-  6369.345  # Radius at 40 latitude. For other latitudes
calculate the radius using the code of Appendix 1 and replace the radius
value above

RADIUS  <-  RADIUS * 1000 # Convert radius from km to m.  g is measured in
m/sec2 so radii must also be in m

phideg <- 40  # Enter any value of latitude as an angle in degrees by
replacing the 40 in this line of code
```

```
phirad <- phideg * pi/180 # Automatic conversion to radians, as required in R

# Now we calculate g for the given latitude and corresponding Earth radius

M <- 5.972e24  # Mass of Earth

G <- 6.674e-11  # Gravitational constant

g <- G * M / RADIUS **2  # Using Newton's Law as an approximation. In fact, the increase in g due to oblateness is not as large as expected from the inverse squared law, and is approximately 0.33 of the observed value. The observed value is larger because the Earth's density increases toward the center. Thus, our calculated values for local gravitational acceleration slightly overestimate the true values. In this simple model g is overestimated by approximately 0.3%.

g  # Prints our estimate of local g on the R console


GEFFRAD  <-  g  -  W**2 * RADIUS * cos(phirad)**2  # Calculate radial component of g – normal component of centrifugal. This is the resultant acceleration we use to calculate weight on the solid earth

GEFFTANG <-  W**2 * RADIUS * cos(phirad) * sin(phirad)  # Calculate tangential component of centrifugal force. On earth, this component is cancelled out by friction but is useful for oceans etc.

GEFF  <- (GEFFRAD**2 + GEFFTANG**2)**0.5  # Calculate vector sum for total effective g

GEFFRAD  # Prints the magnitude of radial component of g for the given angle on the R console

GEFFTANG  # Prints the magnitude of the tangential component of acceleration for the given angle on the R console

GEFF   # Prints the magnitude of effective g for the given latitude on the R console


# Now we calculate the weight of a body at the chosen latitude

MASS <- 70  # Enter a body mass in kg – here it's 70 kg but you can enter your own value

WEIGHT <- MASS * GEFFRAD  # Calculates radial weight using the radial component only. OK for terrestrial measurements because friction cancels out the tangential component

WEIGHT  # Prints the weight found from the radial component
```

```
WEIGHTEFF <- MASS * GEFF  # Calculates 'weight' using total effective g (OK for ocean water etc)

WEIGHTEFF  # Prints the weight using effective g. Because the tangential component is so small, it is essentially the same as that found from the radial component only.
```

APPENDIX 3

R code for calculation of Ɵ, the angle between g and effective g at a given latitude

The angle Ɵ between g and effective g is given by the inverse tan of the ratio of the tangential and radial components of g_{eff}. R uses the **atan()** function to calculate the inverse tan. The angle is usually very small but reaches nearly 0.1° around latitude 45°.

```
rm(list=ls()) # Clears R workspace

# RADIUS    <-  6378.137  # Reminder of radius at equator.

# RADIUS    <-  6356.752  # Reminder of radius at poles.


W  <-  7.292e-5  # Angular velocity of Earth in radians per second

RADIUS  <-  6369.345  # Radius at latitude 40 found using the code of Appendix 1 and used in the code of Appendix 2. Again, for any other latitude use your calculated value from the code of Appendix 1

RADIUS  <-  RADIUS * 1000 # Converting to m

phideg <- 40  # Enter any value of latitude as an angle in degrees by replacing the 40 in this line of code

phirad <- phideg * pi/180  # Automatic conversion to radians, as required in R


# Now we calculate g for the given latitude and corresponding Earth radius

M <- 5.972e24   # Mass of Earth

G <- 6.674e-11  # Gravitational constant

g <- G * M / RADIUS **2  # Using Newton's Law as an approximation got purely gravitational acceleration. In fact, the increase in g due to oblateness is not as large as expected from the inverse squared law, and is approximately 0.33 of the observed value. The observed value is larger
```

```
because the Earth's density increases toward the center. Thus, our
calculated values for local gravitational acceleration slightly
overestimate the true values. In this simple model g is overestimated by
approximately 0.3%.

g  # Prints our estimate of local g on the R console


GEFFRAD  <-  g  -  W**2 * RADIUS * cos(phirad)**2  # Calculate radial
component of g - normal component of centrifugal force. This is the
resultant acceleration we use to calculate weight on the solid earth

GEFFTANG <-  W**2 * RADIUS * cos(phirad) * sin(phirad)  # Calculate
tangential component of centrifugal force. On earth, this component is
cancelled out by friction but is useful for oceans etc.

thetarad <- atan(GEFFTANG / GEFFRAD)

thetarad  # Prints the angle in radians on the R console

theta <- thetarad*180/pi  # Convert from radians back to degrees

theta  # Prints the angle between g and effective g in degrees on the R
console
```

APPENDIX 4

R code for modelling and plotting Earth's radius, effective g and angle theta by latitude

Remember to create a folder to store the pdf graphs of Appendix 4 and navigate your R working directory to that folder. Copy and paste all of the code below into the R workspace together.

```
rm(list=ls()) # Clears the R workspace

# Set up arrays of key variables for calculations

phideg    <- array(100) # Latitudes in degrees
phirad    <- array(100) # Latitudes in radians
NUMERATOR <- array(100) # Used in various calculations
DENOMINATOR <- array(100) # Used in various calculations
RADIUS    <- array(100)  # Earth radius across latitudes
g         <- array(100)  # Gravitational acceleration across latitudes
GEFFRAD   <- array(100)  # Radial component of geff
GEFFTANG  <- array(100)  # Tangential component of geff
GEFF      <- array(100)  # Effective g
THETARAD  <- array(100)  # Theta in radians
THETA     <- array(100)  # Theta in degrees

W  <-  7.292e-5  # Angular velocity of Earth in radians per second
```

```
R1 <- 6378.137  # Established radius in km at the equator

R2 <- 6356.752  # Established radius in km at the poles


# We use a loop to evaluate Earth's radius across the range of latitudes
from one degree to the pole. The equator must be treated separately.

for (k in  1:90) {

phideg[k] <- k

phirad[k] <- phideg[k] * pi/180 # Automatic conversion from degrees to
radians, as required by R

NUMERATOR[k] <- (R1**2 * cos(phirad[k]))**2 + (R2**2 * sin(phirad[k]))**2

DENOMINATOR[k] <- (R1 * cos(phirad[k]))**2 + (R2 * sin(phirad[k]))**2

RADIUS[k] <- (NUMERATOR[k] / DENOMINATOR[k])**0.5

}

RADIUS # Prints out the radii for 1 DEGREE to 90 DEGREES.


# WE NOW CALCULATE THE RADIUS FOR THE EQUATOR (CANNOT BE DONE IN THE ABOVE
LOOP)

NUMERATOREQUATOR <- (R1**2 * cos(0))**2 + (R2**2 * sin(0))**2

DENOMINATOREQUATOR <- (R1 * cos(0))**2 + (R2 * sin(0))**2

RADIUSEQUATOR <-  (NUMERATOREQUATOR / DENOMINATOREQUATOR)**0.5

RADIUSEQUATOR

RADIUS <- c(RADIUSEQUATOR, RADIUS)  # NOW INCLUDING THE EQUATORIAL VALUE

RADIUS



# CREATE A PNG GRAPH OF EARTH'S RADIUS WITH LATITUDE

png("RADIUS_lat.png")

plot(1:91, RADIUS, pch=16, xaxt="n", type = "l", lwd=2, col = "red", xlab =
expression(paste("Latitude (",phi, ")")), ylab = "Earth's Radius (km)",
main= "Variation of Earth's Radius by Latitude", cex.lab=1.3, xlim=
c(0,95), ylim=c(6355, 6380))

axis(1, at=c(0, 10, 20, 30, 40, 50, 60, 70, 80, 90),las=1)

dev.off()

# Now we use a second loop to calculate radial g (GEFFRAD), the tangential
component of the centrifugal acceleration (GEFFTANG) and effective g
(GEFF)I use multiple loops for clarity for other users of the code
```

```
# Now we calculate g and geff across latitudes and corresponding Earth
radii

M <- 5.972e24   # Mass of Earth

G <- 6.674e-11  # Gravitational constant


# WE MUST ELIMINATE THE FIRST ELEMENT FOR THE EQUATOR SO WE CAN USE A LOOP
FOR 1 DEGREE TO 90 DEGREES

RADIUS <- RADIUS[-c(1)]  # Removes the equatorial radius


for (k in  1:90) {

RADIUS[k] <- RADIUS[k]*1000   # Convert km to m

g[k] <- G * M / RADIUS[k]**2  # Purely gravitational acceleration


GEFFRAD[k]  <-  g[k]  -  W**2 * RADIUS[k] * cos(phirad[k])**2  # Calculates
radial component of g - normal component of centrifugal

GEFFTANG[k] <-  W**2 * RADIUS[k] * cos(phirad[k]) * sin(phirad[k])  #
Calculate tangential component of centrifugal force

GEFF[k]   <- (GEFFRAD[k]**2 + GEFFTANG[k]**2)**0.5  # Calculate vector sum
for total effective g

}

g  # Prints the magnitude of g on the R console


GEFFRAD # Prints the magnitude of the radial component of acceleration on
the R console

GEFFTANG   # Prints the magnitude of the tangential component of
acceleration on the R console

GEFF  # Prints the magnitude of effective g on the R console


# CALCULATE EFFECTIVE G FOR THE EQUATOR (CANNOT BE DONE WITHIN THE ABOVE
LOOP WHERE k RUNS FROM 1 TO 90)


RADIUSEQUATOR <-  R1*1000

geq <- G * M / (RADIUSEQUATOR)**2  # Estimate of g at the equator


GEFFRADEQUATOR <-  geq  -  W**2 * RADIUSEQUATOR * cos( 0 )**2  # Calculates
radial component of g - normal component of centrifugal (FOR EQUATOR)

GEFFTANGEQUATOR <-  W**2 * RADIUSEQUATOR * cos( 0 ) * sin( 0 )  # Calculate
tangential component of centrifugal force
```

```
GEFFEQUATOR   <- (GEFFRADEQUATOR **2 + GEFFTANGEQUATOR **2)**0.5  #
Calculate vector sum for total effective g (FOR EQUATOR)


# NOW INCLUDE THE EQUATORIAL VALUES

g <- c(geq, g)

GEFFRAD <- c(GEFFRADEQUATOR, GEFFRAD)

GEFFTANG <- c(RADIUSEQUATOR, GEFFTANG)

GEFF <- c(GEFFEQUATOR, GEFF)

GEFF


# CREATE A PNG GRAPH OF PURELY GRAVITATIONAL ACCELERATION g WITH LATITUDE

png("G_lat.png")

plot(1:91, g, pch=16, xaxt="n", yaxt="n", col = "blue", type = "l", lwd=2,
xlab = expression(paste("Latitude (",phi, ")")), ylab= "", main= "Variation
of Purely Gravitational Acceleration g by Latitude")

axis(1, at=c(0, 10, 20, 30, 40, 50, 60, 70, 80, 90),las=1)

axis(2, at=c(9.76, 9.77, 9.78, 9.79, 9.80, 9.81, 9.82, 9.83, 9.84, 9.85,
9.86, 9.87),las=1)

title(ylab = expression(paste("Purely Gravitational Acceleration g m/",
s^2,")")), line = 2.8, cex.lab=1.0, cex.axis = 0.8)

dev.off()


# CREATE A PNG GRAPH OF RADIAL AND EFFECTIVE g WITH LATITUDE (essentially,
effective g is the same as the radial component)

png("GEFF_lat.png")

plot(1:91, GEFF, pch=16, xaxt="n", yaxt="n", col = "blue", type = "l",
lwd=2, xlab = expression(paste("Latitude (",phi, ")")), ylab= "", main=
"Variation of Radial and Effective g by Latitude")

axis(1, at=c(0, 10, 20, 30, 40, 50, 60, 70, 80, 90),las=1)

axis(2, at=c(9.76, 9.77, 9.78, 9.79, 9.80, 9.81, 9.82, 9.83, 9.84, 9.85,
9.86, 9.87),las=1)

title(ylab = expression(paste("Radial and Effective g (m/", s^2,")")),
line= 2.8, cex.lab=1.0, cex.axis = 0.8)

dev.off()



# CALCULATE THE VARIATION OF THETA WITH LATITUDE USING ANOTHER LOOP FOR
CLARITY
```

```
# Remove the equatorial value of GEFF so we can loop from 1 degree to 90
degrees

GEFF <- GEFF[-c(1)]


for (k in  1:90)  {

THETARAD[k] <- atan((W**2 * RADIUS[k]*cos(phirad[k])* sin(phirad[k])) / (
GEFF[k]   -  W**2 * RADIUS[k]*cos(phirad[k])))

THETARAD  # Prints the angle in radians on the R console

THETA[k] <- THETARAD[k]*180/pi  # Convert from radians back to degrees

}

THETA  # Prints the magnitudes of theta on the R console



# CALCULATE THETA FOR THE EQUATOR (CANNOT BE DONE IN THE ABOVE LOOP).
CALCULATION IN FULL, THOUGH IT MUST BE ZERO BECAUSE OF THE SINE FUNCTION
TAKING THE VALUE ZERO AT ZERO LATITUDE


THETAEQUATOR <- atan((W**2 * RADIUSEQUATOR * cos( 0 )* sin( 0 )) /
(GEFFEQUATOR  -  W**2 * RADIUSEQUATOR *cos( 0 )))

# THETARAD  # Prints the angle in radians on the R console

THETAEQUATOR <-  THETAEQUATOR *180/pi  # Convert from radians back to
degrees

THETA <- c(THETAEQUATOR, THETA)

THETA


# CREATE A PNG GRAPH OF THETA WITH LATITUDE

png("THETA_lat.png")

plot(1:91, THETA, pch=16, xaxt="n", yaxt="n", col = "darkgreen", type =
"l", lwd=2,  xlab = expression(paste("Theta (",theta, ")")),
ylab= "", main= "Variation of Theta for Effective g by Latitude")

axis(1, at=c(0, 10, 20, 30, 40, 50, 60, 70, 80, 90),las=1)
axis(2, at=c(0.0, 0.02, 0.04, 0.06, 0.08, 0.1),las=1)

title(ylab = expression(paste("Theta for Effective g in degrees")), line =
2.8, cex.lab=1.0, cex.axis = 0.8)

dev.off()
```